Exercise Set 3
Suggested Solutions

1. Solution scripts

   a)
```
t=1790:2000;
p=310273000./(1+exp(-0.0353.*(t-1913.25)));
plot(t,p,'k--');
xlabel('Year $t$','Interpreter', 'latex');
ylabel('Population $P(t)$','Interpreter', 'latex');
title('US population')
```

   b)
```
subplot(1,2,1)
[x,y]=meshgrid(-5:.5:5,-10:.5:10);
z=x.^2+2*sin(y);
surf(x,y,z);
xlabel('x-axis'); ylabel('y-axis'); zlabel('z-axis');

subplot(1,2,2)
contour(x,y,z)
xlabel('x-axis'); ylabel('y-axis');
axis square;
```

   c)
```
clf
N=7; k=50; p=0.25; x=0:1:N;
probs = binopdf(x,N,p);
hold on; bar(x,probs)
xap=0:0.1:7; approx = pdf('normal', xap, N*p, N*p*(1-p));
plot(xap,approx)
```

2. Examples of doing linear algebra with Matlab.

   a)
```
A=[25 -25 15 1; 0 1 1 0; 1 -5 -3 -4; 1 0 0 -2];
b=[7 1 -16 0];
rank(A)
sol=A\b'; %or sol=inv(A)*b'
```

   b) The solution by hand is $x_1 = 0$, $x_2 = (4/9)10^{10}$, $x_3 = 10^{10}$.

```
format long;
A=[1e-10 0.9 -0.4; 0 0.9 -0.4; 0  0 1e-10];
```

```
b=[0 0 1];
sol=A\b';
%In this example having elements of different magnitude
%makes the matrix "badly scaled".
%This can be measured by condition number,
%which measures the sensitivity to errors
cond(A)
%this is relatively large indicating
%that numerical errors are likely
```

c)
```
A=[0.3 0.6; 0.7 0.4];
[vecs,vals]=eig(A);
%identify the colums of vecs corresponding to
%eigenvalue 1 and scale it to get a distribution
stadist=vecs(:,2)./sum(vecs(:,2))
```

3. Functions.

a)
```
function output=discount(x,delta)
%input should be a row vector
n=length(x);
pows=[0:1:n-1];
output=sum(delta.^pows.*x);
```

b)
```
function PVA=annuity(A,r,n)
%Exercise 3
PVA=A*((1 - (1 / (1 + r)^n)) / r);
%Use the following call to find the indifference point
%A=50000;n=20;fzero(@(r) annuity(A,r,n)-500000,0.07)
```

4. Simulation of asset prices.

```
function [means,vars]=assetsim(mu,sigma,p0,A,n,k)
%Exercise 4
%example of a call
%[means,vars]=assetsim(0.04,0.3,100,100,20,5000)
dt = 1;%1/n;
%initializations
x=zeros(n,1); %number of assets at each time instant
p(1)=p0; %initial price
```

```
x(1)=A/p0; %initial holdings
%final wealths with two different investment strategies
wealth1=zeros(k,1);
wealth2=wealth1;
r=mu; %interest rate
pva=annuity(A,r,n-1)+A;
%present value of annuity paid throughout
%n periods plus initial period

%loop over simulations
for j = 1:k % perform k random walks
for i = 1:n-1
dX = randn*dt;
dP = p(i)*(mu*dt + sigma*dX);
p(i+1) = p(i)+dP;
x(i+1) = x(i)+A/p(i+1);
end

wealth1(j)= x(end).*p(end);
wealth2(j)=(pva/p(1)).*p(end);
%number of assets obtained in period 1 * final price
end
means=[mean(wealth1) mean(wealth2)];
vars=[var(wealth1) var(wealth2)];
```

5. Dynare example. The model part of the file is:

```
model;
x = x(+1) - 1/sig*(i-pi(+1)-r);
pi= bet*pi(+1)+ kappa*x ;
r = rho+sig*psi*(rhoa-1)*a;
i = rho + phipi*pi + phix*x ;
a = rhoa*a(-1)+ae;
end;
```